Solving Equations

Show all work required to solve each equation.

Example: $3(5x - 3x) + 5 = 47$

$$3(2x) + 5 = 47$$
$$6x + 5 = 47$$
$$6x = 42$$
$$x = 7$$

a) $2(7x - 3x) + 4 = 28$

b) $4x + 18 = 7(x + 3)$

c) $3x + 2(x + 2) = 13 - (2x + 2)$

d) $4(3x + x) + 7 - 5x = 8 + (-5)(5x - 6x) + 23$

Multiplying Binomials

Use a distribution process (you may know it as "FOIL")

Example: #1 $(x - 4)(x - 5)$ distribute the first x over $(x - 5)$ and repeat with the -4

$$x^2 - 5x - 4x + 20$$
$$x^2 - 9x + 20$$

Example #2 $(x + 5)^2$ define

$(x + 5)(x + 5)$ now distribute

$$x^2 + 5x + 5x + 25$$
$$x^2 + 10x + 25$$

a) $(x - 3)(x - 4)$

b) $(5x + 4)(7x + 1)$

c) $(x - 3)(x + 5)$

d) $(4x - 3y)^2$

e) $(c^2d^2 + 5)(c^2d^2 - 5)$

Simplifying Radicals

There are many different techniques available to simplify radical expressions. If the technique demonstrated below is not what you know, feel free to use any other technique (provided it applies legitimate mathematics).

Example: $\sqrt{20} = \sqrt{(4)(5)} = (\sqrt{4})(\sqrt{5}) = 2\sqrt{5}$

a) $\sqrt{32}$

b) $\sqrt{50}$

c) $\sqrt{50}$

d) $\sqrt{108}$

e) $7\sqrt{32}$

f) $\sqrt{72a^3c^4}$

When a radical expression occurs in a rational expression, be sure to follow the basic rules for simplifying fractions.

Example: $\dfrac{10 \pm \sqrt{80}}{2} = \dfrac{10 \pm 4\sqrt{5}}{2} = \dfrac{10}{2} \pm \dfrac{4\sqrt{5}}{2} = 5 \pm 2\sqrt{5}$

a) $\dfrac{12 \pm \sqrt{45}}{3}$

b) $\dfrac{-14 \pm \sqrt{98}}{7}$

Factoring

Start with looking for the greatest common factor (GCF)

Example: $5x^4 + 15x^3$ 5 and x^3 are factors of both terms

$5x^3 (x + 3)$

a) $6x^3 - 9x^2$

b) $12a^2 - 18ab$

c) $8xy + 8y^2 - 8yz$

d) $63x^4 + 81x^3 - 72x^2$

If there is no GCF, need to look for a pair of binomials that will "FOIL" to get the trinomial.

Example: $x^2 + 9x + 18$ look for a pair of factors of 18 whose sum is 9

$(x + 3) (x + 6)$

then check using "FOIL"

e) $x^2 + 19x + 18$

f) $x^2 - 17x + 16$

g) $x^2 + 3x - 28$

h) $x^2 - 16$

When the leading coefficient is something other than 1, you must look for pairs of binomials with coefficients of x whose product is the quadratic term

Example: $12x^2 + 17x + 6$

look for pairs of factors of 12 (the leading coefficient) and pairs of factors of 6 (the constant)

place those values into binomials to utilize a guess and check process

$(6x + 3) (2x + 2)$ ---- "FOIL" produces $12x^2 + 12x + 6x + 6$

$12x^2 + 18x + 6$ not correct

$(3x + 3) (4x + 2)$ ---- "FOIL" produces $12x^2 + 12x + 6x + 6$

$12x^2 + 18x + 6$ not correct

$(4x + 3) (3x + 2)$ ---- "FOIL" produces $12x^2 + 8x + 9x + 6$

$12x^2 + 17x + 6$ correct

i) $4x^2 + 5x + 1$

j) $2x^2 - 5x - 3$

k) $4y^2 - 4y + 1$

l) $3x^2 - 2x - 1$

Solving Quadratic Equations

The Zero Product Property states that if the product of two values is zero, then one of the two values must be zero. We can apply this to solving equations. If an expression can be factored into two binomials, and the product of those binomials is zero, then one or both of the binomials must equal zero.

Example: $x^2 - 8x - 20 = 0$ factor (note that it needs to equal zero before factoring)

$(x - 10)(x + 2) = 0$ apply the Zero Product Property

$x - 10 = 0$ OR $x + 2 = 0$

$x = 10$ $x = -2$

a) $x^2 - 7x + 10 = 0$

b) $x^2 - 5x - 10 = 4$

c) $x^2 - 7x = -12$

d) $x^2 + 3x - 72 = 4x$

If the expression is not factorable, then the Quadratic Formula can be used to solve the equation. Recall the formula is: for an expression of the type $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: $x^2 + 4x - 8 = 0$

$a = 1$ $b = 4$ $c = -8$

$$\text{so, } x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-8)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 + 32}}{2}$$

$\sqrt{16}\sqrt{3}$

$$x = \frac{-4 \pm \sqrt{48}}{2} = \frac{-4 \pm 4\sqrt{3}}{2} = -2 \pm 2\sqrt{3}$$

e) $x^2 - 16x + 19 = 0$

f) $9x^2 - 36x - 2 = 0$

g) $5x^2 - 3x - 2 = 0$

Solving Systems of Equations

There are many ways to solve a system of equations. The most commonly used approaches are substitution and elimination.

Example of substitution: $y = 4x - 1$
$3x + 2y = 9$

since you know an expression equivalent to y, make the substitution

$3x + 2(4x - 1) = 9$
$3x + 8x - 2 = 9$
$11x - 2 = 9$
$11x = 11$
$x = 1$

now substitute to find y
$y = 4(1) - 1$
$y = 3$

Example of elimination: $2x + y = 7$
$3x - y = -12$

add the equations together to eliminate a variable with opposite coefficients (you may have to multiply an through in order to get opposite coefficients)

$5x + 0y = -5$
$5x = -5$
$x = -1$

now substitute to find y
$2(-1) + y = 7$
$-2 + y = 7$
$y = 9$

a) $x = y - 3$
$5x + 3y = 1$

b) $y = -4x$
$6x + y = 6$

c) $-2x + 3y = 17$
$2x + y = 3$

d) $3x + 5y = 26$
$2x - y = 13$

e) $3x + 4y = 11$
$5x + 2y = -5$

f) $4x + 3y = 7$
$2x - 9y = 35$

Introduction to Logic

Much of the study of geometry is combining together previously known information and then drawing a conclusion. For each set of statements below, write a third statement that is true based on the information given. (This is known as making a conjecture.)

a) Abigail is a freshman at Bartlett High School.
All Bartlett High School freshman take biology.

b) Clouds produce rain.
It is raining.

c) It takes hard work to be a good student.
Anthony works hard.

d) If the sun shines, then the plants will grow.
The sun is shining.

Below is a set of statements about the situation indicated. Combine the information together to respond as indicated.

a) A drummer, guitarist, and keyboard player named Amy, Bob, and Carla are in a band. Use the clues presented here to determine which instrument each plays.
Carla and the drummer wear different colored shirts.
The keyboard player is older than Bob.
Amy, the youngest band member, lives next door to the guitarist.

b) Three red hats and three blue hats are packed in three boxes, with two hats to a box. The boxes are all labeled incorrectly. To determine what each box actually contains, you may select one hat from one box, without looking at the contents of the box. Explain how this will allow you to determine the contents of each box. (Hint: List all possible solutions; then use logic to solve.)

c) Alan, Ben, and Cal are seated in order, Alan is behind Ben who is behind Cal. They all have their eyes closed. Three hats are placed on their heads from a box they know contains 3 red and 2 blue hats. They open their eyes and look forward.
Alan says, "I cannot deduce what color hat I'm wearing."
Hearing that, Ben says, "I cannot deduce what color I'm wearing either."
Cal then says, "I know what color I am wearing!"
How does Cal know what color his hat is?